**Notations:**

: response measurement for subject i at time

: n by 1 vector of response measurements for subject i

: explanatory variable k observed at time (sometimes we use )

: p by 1 vector of explanatory variables at time

: n by p matrix of covariates for subject i

**Properties of   and S:**

Define   as an n by 1 vector and as an n by n matrix

where is a n by 1 vector of independent and identically distributed samples from a multivariate distribution with mean (n by 1) and covariance matrix (n by n). Then and S are unbiased for and , respectively.

In particular, and :

(1) , and

(2) First, . Therefore, we have .

Second, we have

Third, we have

Therefore, we have

**Multivariate normal distribution:**

If Y (n by 1) ~ MVN (), then the density function for Y is given by

Because

Therefore

One can verify that , and .

**Malhalanobis Distance:**

-- In the univariate setting, standardized residuals are often used to measure the distance of value from the mean

-- In the multivariate setting, we define

And the observed value

**Transformation of MVN random variables:**

When , then also follows a MVN distribution, where A is an r by n matrix; C is an r by 1 vector; Z is an r by 1 vector.

Also,

The above holds because -> -> -> is a summation of the square of n random variables that follow a standard normal distribution.

Marginals of MVN:

If , then .

Joint MVN implies marginal normality. But the reverse is not true.

Conditional of MVN:

We are interested in split the Y vector into part: , where Y1 is a q by 1 vector and Y2 is an r by 1 vector, q+r=n.

Then the conditional distribution of Y1|Y2 is also a MVN:

Where:

Distribution of sample mean and variance

If , then

Where a random n by n matrix W is said to follow an n-dimensional Wishart distribution , with N degree of freedom and parameter , if W can be represented as , where Xj, j=1, …, N, are i.i.d. from MVN (0, ). Provided that N>n, the density function of W is given by

For n=1, with, then .

In addition, .